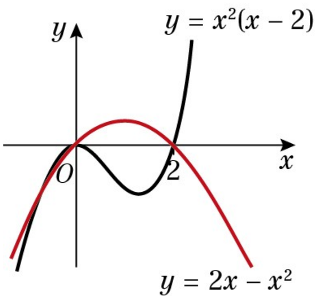
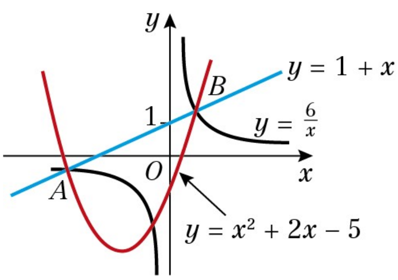
Chapter review 4

1 a $y = x^2(x - 2)$
$0 = x^2(x - 2)$
So $x = 0$ or $x = 2$
The curve crosses the x-axis at (2, 0) and touches it at (0, 0).
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$
$y = 2x - x^2$
$\quad = x(2 - x)$
As $a = -1$ is negative, the graph has a shape and a maximum point.
$0 = x(2 - x)$
So $x = 0$ or $x = 2$
The curve crosses the x-axis at (0, 0) and (2, 0).



b $x^2(x - 2) = x(2 - x)$
$x^2(x - 2) - x(2 - x) = 0$
$x^2(x - 2) + x(x - 2) = 0$
$x(x - 2)(x + 1) = 0$
So $x = 0$, $x = 2$ or $x = -1$
Using $y = x(2 - x)$:
when $x = 0$, $y = 0 \times 2 = 0$
when $x = 2$, $y = 2 \times 0 = 0$
when $x = -1$, $y = (-1) \times 3 = -3$
The points of intersection are (0, 0), (2, 0) and (−1, −3).

2 a $y = \dfrac{6}{x}$ is like $y = \dfrac{1}{x}$.
$y = 1 + x$ is a straight line.

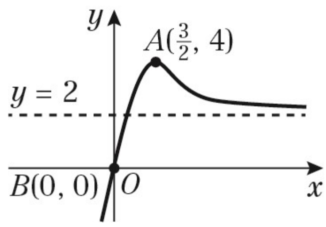


2 b $\dfrac{6}{x} = 1 + x$
$6 = x + x^2$
$0 = x^2 + x - 6$
$0 = (x + 3)(x - 2)$
So $x = 2$ or $x = -3$
Using $y = 1 + x$:
when $x = 2$, $y = 1 + 2 = 3$
when $x = -3$, $y = 1 - 3 = -2$
So A is (−3, −2) and B is (2, 3).

c Substituting the points A and B into $y = x^2 + px + q$:
A: $-2 = 9 - 3p + q$ **(1)**
B: $3 = 4 + 2p + q$ **(2)**
(1) − **(2)**:
$-5 = 5 - 5p$
$p = 2$
Substituting in (**1**):
$-2 = 9 - 6 + q$
$q = -5$

d $y = x^2 + 2x - 5$
As $a = 1$ is positive, the graph has a shape and a minimum point.
$y = (x + 1)^2 - 6$
So the minimum is at (−1, −6).

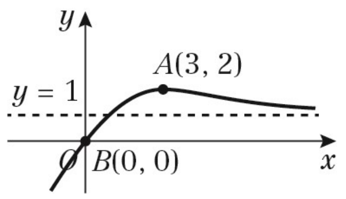
3 a f(2x) is a stretch with scale factor $\frac{1}{2}$ in the x-direction.



$A'\left(\frac{3}{2}, 4\right)$, $B'(0, 0)$
The asymptote is $y = 2$.

b $\frac{1}{2}$f(x) is a stretch with scale factor $\frac{1}{2}$ in the y-direction.

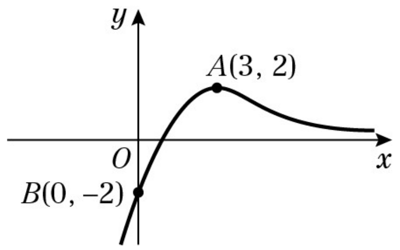
3 b



$A'(3, 2)$, $B'(0, 0)$
The asymptote is $y = 1$.

c $f(x) - 2$ is a translation by $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$,

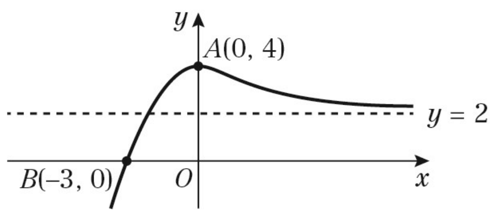
or two units down.



$A'(3, 2)$, $B'(0, -2)$
The asymptote is $y = 0$.

d $f(x + 3)$ is a translation by $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$,

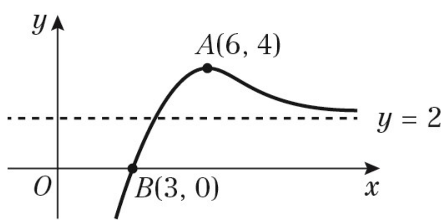
or three units to the left.



$A'(0, 4)$, $B'(-3, 0)$
The asymptote is $y = 2$.

e $f(x - 3)$ is a translation by $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$,

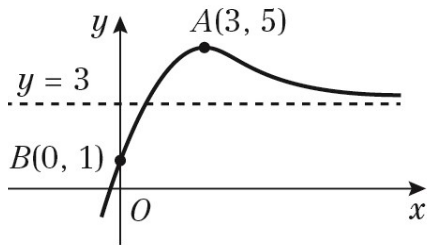
or three units to the right.



e $A'(6, 4)$, $B'(3, 0)$
The asymptote is $y = 2$.

f $f(x) + 1$ is a translation by $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$,

or one unit up.



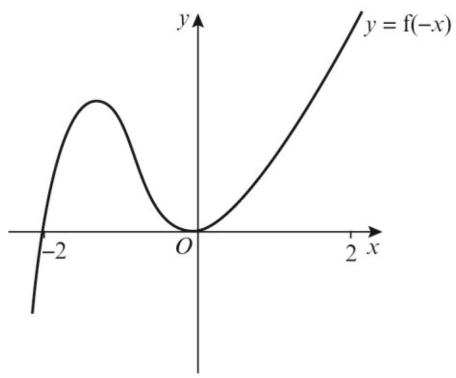
$A'(3, 5)$, $B'(0, 1)$
The asymptote is $y = 3$.

4

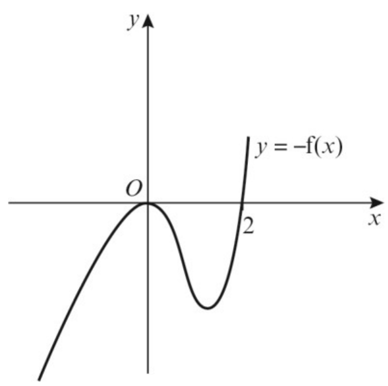
$$2 = 5 + 2x - x^2$$
$$x^2 - 2x - 3 = 0$$
$$(x - 3)(x + 1) = 0$$

So $x = -1$ or $x = 3$
The points of intersection are $A(-1, 2)$, $B(3, 2)$.

5 a $f(-x)$ is a reflection in the y-axis.

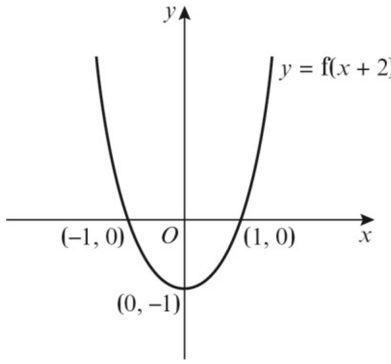


b $-f(x)$ is a reflection in the x-axis.

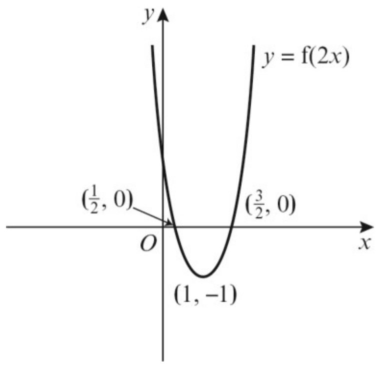


6 a Let $y = a(x - p)(x - q)$.
Since (1, 0) and (3, 0) are on the curve then $p = 1$ and $q = 3$.
So $y = a(x - 1)(x - 3)$
Using $(2, -1)$:
$-1 = a(1)(-1)$
$a = 1$
So $y = (x - 1)(x - 3) = x^2 - 4x + 3$

b i $f(x + 2) = (x + 1)(x - 1)$, or a translation by $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$, or two units to the left.



ii $f(2x) = (2x - 1)(2x - 3)$, or a stretch with scale factor $\frac{1}{2}$ in the x-direction.



7 a $f(x) = (x - 1)(x - 2)(x + 1)$
When $x = 0$, $y = (-1) \times (-2) \times 1 = 2$
So the curve crosses the y-axis at (0, 2).

b $y = af(x)$ is a stretch with scale factor a in the y-direction.
The y-coordinate has multiplied by -2, therefore $y = -2f(x)$.
$a = -2$

7 c $f(x) = (x - 1)(x - 2)(x + 1)$
$0 = (x - 1)(x - 2)(x + 1)$
So $x = 1$, $x = 2$ or $x = -1$
The curve crosses the x-axis at (1, 0), (2, 0) and $(-1, 0)$.
$y = f(x + b)$ is a translation b units to the left.
For the point (0, 0) to lie on the translated curve, either the point (1, 0), (2, 0) or $(-1, 0)$ has translated to the point (1, 0).
For the coordinate (1, 0) to be translated to (0, 0), $b = 1$.
For the coordinate (2, 0) to be translated to (0, 0), $b = 2$.
For the coordinate $(-1, 0)$ to be translated to (0, 0), $b = -1$.
$b = -1$, $b = 1$ or $b = 2$

8 a i $y = f(3x)$ is a stretch with scale factor $\frac{1}{3}$ in the x-direction. Find $\frac{1}{3}$ of the x-coordinate.
P is transformed to $\left(\frac{4}{3}, 3\right)$.

ii $\frac{1}{2}y = f(x)$
$y = 2f(x)$, which is a stretch with scale factor 2 in the y-direction.
P is transformed to (4, 6).

iii $y = f(x - 5)$ is a translation by $\begin{pmatrix} 5 \\ 0 \end{pmatrix}$, or five units to the right.
P is transformed to (9, 3).

iv $-y = f(x)$
$y = -f(x)$, which is a reflection of the curve in the x-axis.
$(4, -3)$

v $2(y + 2) = f(x)$
$y = \frac{1}{2}f(x) - 2$, which is a stretch with scale factor $\frac{1}{2}$ in the y-direction and then a translation by $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$, or two units down.
P is transformed to $\left(4, -\frac{1}{2}\right)$.

8 b $P(4, 3)$ is transformed to $(2, 3)$.
Either the x-coordinate has halved, which is a stretch with scale factor $\frac{1}{2}$ in the x-direction, or it has had 2 subtracted from it, which is a translation by $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$, or two units to the left.

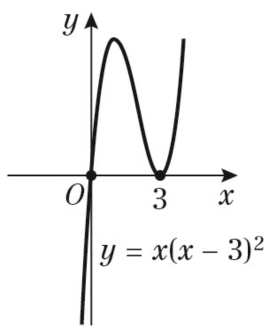
So the transformation is $y = \mathrm{f}(2x)$ or $y = \mathrm{f}(x + 2)$.

c i $P(4, 3)$ is translated to the point $(8, 6)$.
The x-coordinate of P has 4 added to it and the y-coordinate has 3 added to it.
$y = \mathrm{f}(x - 4) + 3$

ii $P(4, 3)$ is stretched to the point $(8, 6)$.
The x-coordinate of P has doubled and the y-coordinate has doubled.
$y = 2\mathrm{f}\left(\frac{1}{2}x\right)$

9 a $x^3 - 6x^2 + 9x$
$= x(x^2 - 6x + 9)$
$= x(x - 3)^2$

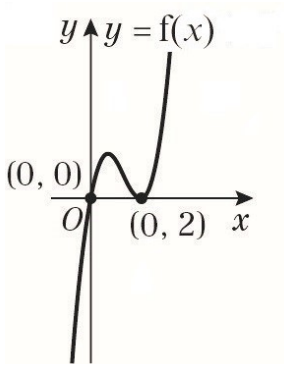
b $y = x(x - 3)^2$
$0 = x(x - 3)^2$
So $x = 0$ or $x = 3$
The curve crosses the x-axis at $(0, 0)$ and touches it at $(3, 0)$.
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



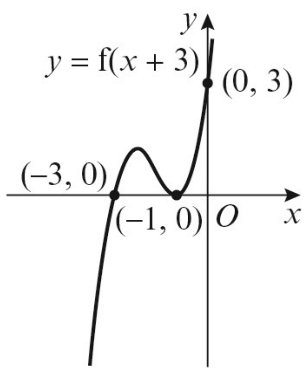
c $y = (x - k)^3 - 6(x - k)^2 + 9(x - k)$ is a translation of the curve $y = x^3 - 6x^2 + 9x$ by $\begin{pmatrix} k \\ 0 \end{pmatrix}$, or k units to the right.

For the point $(-4, 0)$ to lie on the translated curve, either the point $(0, 0)$ or $(3, 0)$ has translated to the point $(-4, 0)$.
For the coordinate $(0, 0)$ to be translated to $(-4, 0)$, $k = -4$.
For the coordinate $(3, 0)$ to be translated to $(-4, 0)$, $k = -7$.
$k = -4$ or $k = -7$

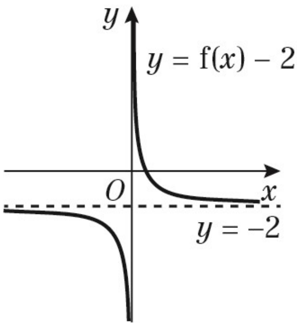
10 a $y = x(x - 2)^2$
$0 = x(x - 2)^2$
So $x = 0$ or $x = 2$
The curve crosses the x-axis at $(0, 0)$ and touches it at $(2, 0)$.
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



b $y = \mathrm{f}(x + 3)$ is a translation by vector $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$ of $y = \mathrm{f}(x)$, or three units to the left.
So the curve crosses the x-axis at $(-3, 0)$ and touches it at $(-1, 0)$.
When $x = 3$, $\mathrm{f}(x) = 3(3 - 2)^2 = 3$
So $\mathrm{f}(x + 3)$ crosses the y-axis at $(0, 3)$.



11 a $y = \mathrm{f}(x) - 2$ is a translation by $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$, or two units down.



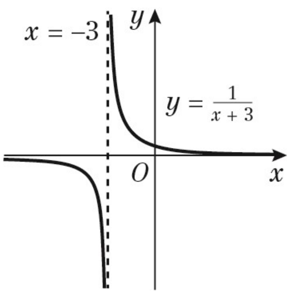
The horizontal asymptote is $y = -2$.
The vertical asymptote is $x = 0$.

b From the sketch, the curve crosses the x-axis.

$$y = \mathrm{f}(x) - 2$$
$$= \frac{1}{x} - 2$$
$$0 = \frac{1}{x} - 2$$
$$x = \tfrac{1}{2}$$

So the curve cuts the x-axis at $\left(\tfrac{1}{2}, 0\right)$.

c $y = \mathrm{f}(x + 3)$ is a translation by $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$, or three units to the left.



d The horizontal asymptote is $y = 0$.
The vertical asymptote is $x = -3$.

$$y = \mathrm{f}(x + 3)$$
$$= \frac{1}{x+3}$$

When $x = 0$, $y = \tfrac{1}{3}$

So the curve cuts the y-axis at $\left(0, \tfrac{1}{3}\right)$.

Challenge

$R(6, -4)$

$y = \mathrm{f}(x + c) - d$ is a translation by $\begin{pmatrix} -c \\ 0 \end{pmatrix}$, or c units to the left and a translation by $\begin{pmatrix} 0 \\ -d \end{pmatrix}$, or d units down.

So R is transformed to $(6 - c, -4 - d)$.